Chapter 12 – Independent Demand Inventory Management

Inventories in the Supply Chain

Independent vs. Dependent Demand

- Independent demand items are finished goods or other items sold to someone outside the company
- Dependent demand items are materials or component parts used in the production of another item (e.g., finished product)

Types of Inventory: How Inventory is Used

- Anticipation or seasonal inventory
- Safety stock: buffer demand fluctuations
- Lot-size or cycle stock: take advantage of quantity discounts or purchasing efficiencies
- Pipeline or transportation inventory
- Speculative or hedge inventory protects against some future event, e.g. labor strike
- Maintenance, repair, and operating (MRO) inventories

Objectives of Inventory Management

- Provide acceptable level of customer service (on-time delivery)
- Allow cost-efficient operations
- Minimize inventory investment

Relevant Inventory Costs

Item Cost	**Cost per item plus any other direct costs associated with getting the item to the plant**
Holding Costs	**Capital, storage, and risk cost typically stated as a % of the unit value,** **e.g. 15-25%**
Ordering Cost	**Fixed, constant dollar amount incurred for each order placed**
Shortage Costs	**Loss of customer goodwill, back order handling, and lost sales**

Order Quantity Strategies

Lot-for-lot	Order exactly what is needed for the next period
Fixed-order quantity	Order a predetermined amount each time an order is placed
Min-max system	When on-hand inventory falls below a predetermined minimum level, order enough to refill up to maximum level
Order *n* periods	Order enough to satisfy demand for the next ***n*** periods

Examples of Ordering Approaches

Lot for Lot Example								
	1	2	3	4	5	6	7	8
Requirements	70	70	65	60	55	85	75	85
Projected-on-Hand (30)	0	0	0	0	0	0	0	
Order Placement	40	70	65	60	55	85	75	85

Fixed Order Quantity Example with Order Quantity of 200								
	1	2	3	4	5	6	7	8
Requirements	70	70	65	60	55	85	75	85
Projected-on-Hand (30)	160	90	25	165	110	25	150	65
Order Placement	200			200			200	

Min-Max Example with min.= 50 and max.= 250 units								
	1	2	3	4	5	6	7	8
Requirements	70	70	65	60	55	85	75	85
Projected-on-Hand (30)	180	110	185	125	70	165	90	165
Order Placement	220		140			180		160

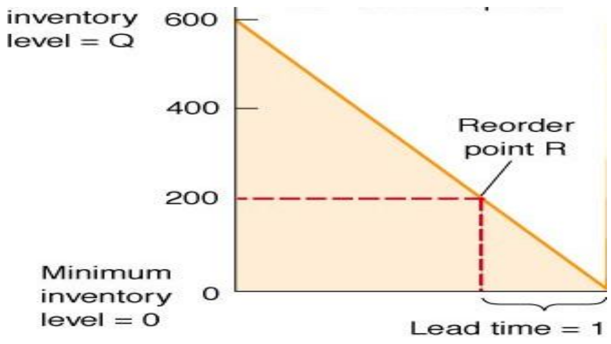
Order *n* Periods with n = 3 periods								
	1	2	3	4	5	6	7	8
Requirements	70	70	65	60	55	85	75	85
Projected-on-Hand (30)	135	65	0	140	85	0	85	0
Order Placement	175			200			160	

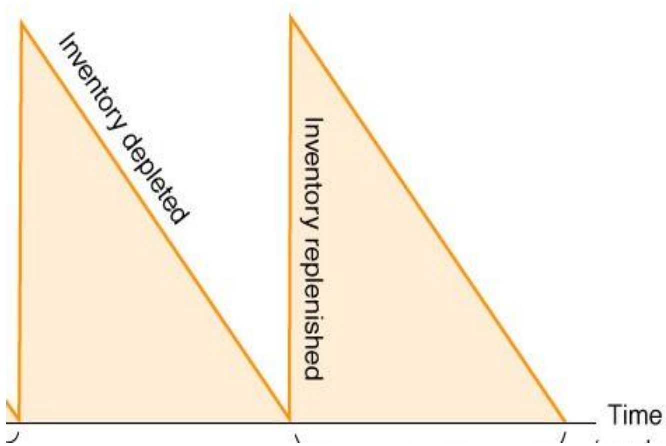
Three Mathematical Models for Determining Order Quantity

- **Economic Order Quantity (EOQ or Q System)**
  - An optimizing method used for determining order quantity and reorder points
  - Part of **continuous review system** which tracks on-hand inventory each time a withdrawal is made
- **Economic Production Quantity (EPQ)**
  - A model that allows for incremental product delivery
- **Quantity Discount Model**
  - Modifies the EOQ process to consider cases where quantity discounts are available

Economic Order Quantity

- **EOQ Assumptions:**
  - Demand is known & constant - no safety stock is required
  - Lead time is known & constant
  - No quantity discounts are available
  - Ordering (or setup) costs are constant
  - All demand is satisfied (no shortages)
  - The order quantity arrives in a single shipment





EOQ: Total Cost Equation

$$TC_{EOQ} = \left(\frac{D}{Q}S\right) + \left(\frac{Q}{2}H\right)$$

Where

TC = total annual cost

D = annual demand

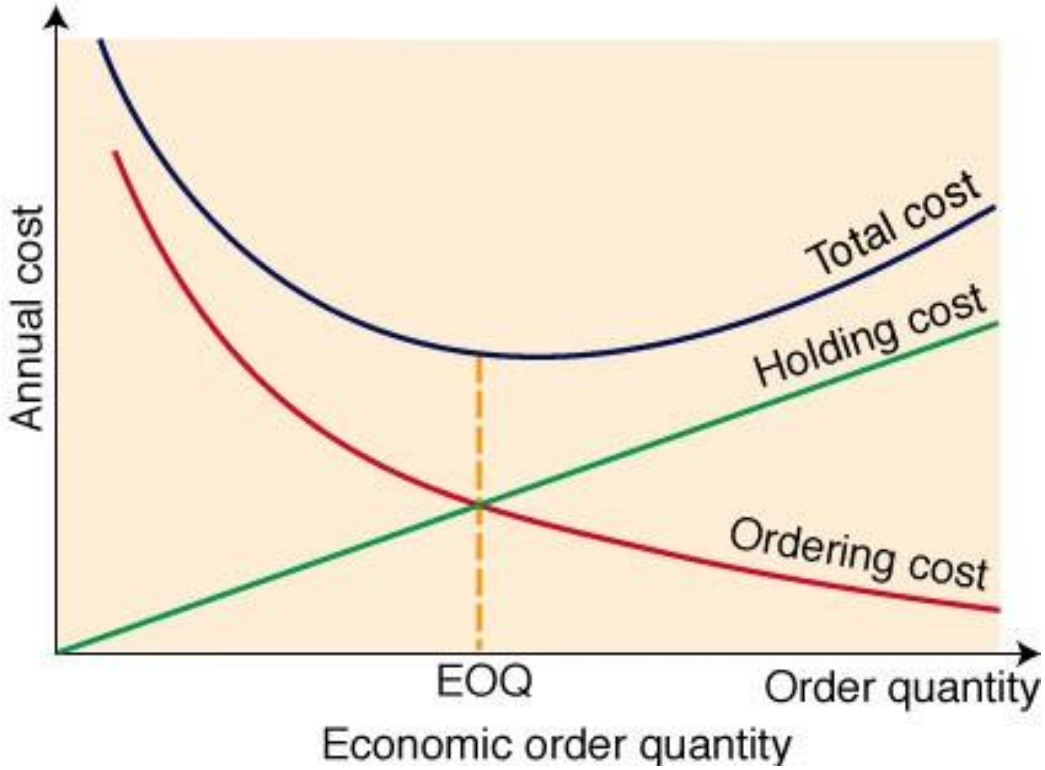
Q = quantity to be ordered

H = annual holding cost

S = ordering or setup cost

EOQ Total Costs

Total annual costs = annual ordering costs + annual holding costs



The EOQ Formula

Minimize the TC by ordering the EOQ:

$$EOQ = \sqrt{\frac{2DS}{H}}$$

When to Order: The Reorder Point

- Without safety stock:

$$R = dL$$

where R = reorder point in units

d = daily/weekly demand in units

L = lead time in days/weeks

- With safety stock:

$$R = dL + SS$$

where SS = safety stock in units

EOQ Example

- Weekly demand = 240 units
- No. of weeks per year = 52
- Ordering cost = $50
- Unit cost = $15
- Annual carrying charge = 20%
- Lead time = 2 weeks

EOQ Example Solution

$$D = 52 \times 240 = 12{,}480\, units\ /\ year$$

$$H = 0.2 \times 15 = \$3\ per\ unit\ per\ year$$

$$Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 12{,}480 \times 50}{3}} = 644.98 \cong 645\, units$$

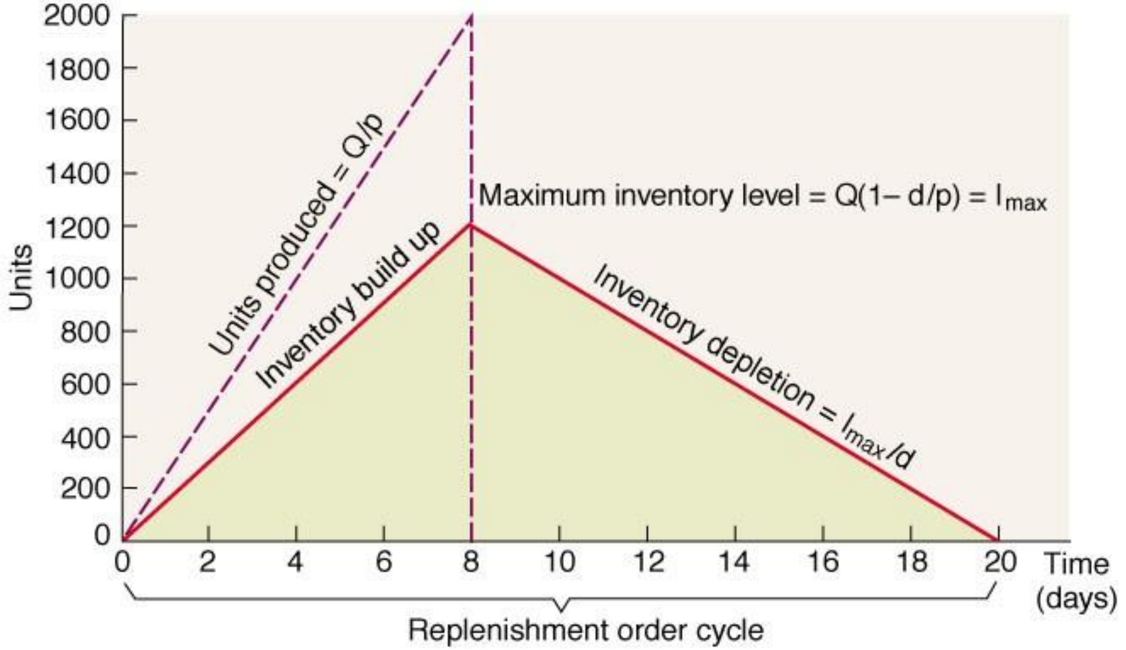
$$TC = \left(\frac{D}{Q} S\right) + \left(\frac{Q}{2} H\right) = \left(\frac{12{,}480}{645} \times 50\right) + \left(\frac{645}{2} \times 3\right)$$

$$= 967.44 + 967.5 = \$1{,}934.94$$

$$R = dL = 240 \times 2 = 480\, units$$

EPQ (Economic Production Quantity) Assumptions

- Same as the EOQ except: inventory arrives in increments & is drawn down as it arrives



Order quantity 2000 units
Daily demand (d) = 100 units
Daily production (p) = 250 units

EPQ Equations

- Adjusted total cost: $$TC_{EPQ} = \left(\frac{D}{Q}S\right) + \left(\frac{I_{MAX}}{2}H\right)$$
- Maximum inventory: $$I_{MAX} = Q\left(1 - \frac{d}{p}\right)$$
- Adjusted order quantity: $$EPQ = \sqrt{\frac{2DS}{H\left(1 - \frac{d}{p}\right)}}$$

EPQ Example

- Annual demand = 18,000 units
- Production rate = 2500 units/month
- Setup cost = $800
- Annual holding cost = $18 per unit
- Lead time = 5 days
- No. of operating days per month = 20

EPQ Example Solution

$$d = \frac{18{,}000}{12} = 1500\, units / month; \;\; p = 2500\, units / month$$

$$Q = \sqrt{\frac{2DS}{H\left(1 - \frac{d}{p}\right)}} = \sqrt{\frac{2 \times 18{,}000 \times 800}{18 \times \left(1 - \frac{1500}{2500}\right)}} = 2000\, units$$

$$I_{MAX} = Q\left(1 - \frac{d}{p}\right) = 2000 \times \left(1 - \frac{1500}{2500}\right) = 800\, units$$

$$TC = \left(\frac{D}{Q} S\right) + \left(\frac{I_{MAX}}{2} H\right) = \left(\frac{18{,}000}{2000} \times 800\right) + \left(\frac{800}{2} \times 18\right)$$

$$= 7{,}200 + 7{,}200 = 14{,}400$$

EPQ Example Solution (cont.)

- The reorder point:

$$R = dL = \frac{1500}{20} \times 5 = 375 \, units$$

- With safety stock of 200 units:

$$R = dL + SS = \frac{1500}{20} \times 5 + 200 = 575 \, units$$

Quantity Discount Model Assumptions

- Same as the EOQ, except:
  - Unit price depends upon the quantity ordered
- Adjusted total cost equation:

$$TC_{QD} = \left(\frac{D}{Q} S \right) + \left(\frac{Q}{2} H \right) + PD$$

Quantity Discount Procedure

- Calculate the EOQ at the lowest price
- Determine whether the EOQ is feasible at that price
  - Will the vendor sell that quantity at that price?
- If yes, stop – if no, continue
- Check the feasibility of EOQ at the next higher price

- Continue to the next slide ...

QD Procedure (continued)

- Continue until you identify a feasible EOQ
- Calculate the total costs (including total item cost) for the feasible EOQ model
- Calculate the total costs of buying at the minimum quantity required for each of the cheaper unit prices
- **Compare the total cost of each option & choose the lowest cost alternative**
- **Any other issues to consider?**

QD Example

- Annual Demand = 5000 units
- Ordering cost = $49
- Annual carrying charge = 20%
- Unit price schedule:

Quantity	Unit Price
0 to 999	$5.00
1000 to 1999	$4.80
2000 and over	$4.75

QD Example Solution

- Step 1

$$Q_{P=\$4.75} = \sqrt{\frac{2\times 5{,}000\times 49}{0.2\times 4.75}} = 718 \ \left(\textit{not feasible}\right)$$

$$Q_{P=\$4.80} = \sqrt{\frac{2\times 5{,}000\times 49}{0.2\times 4.80}} = 714 \ \left(\textit{not feasible}\right)$$

$$Q_{P=\$5.00} = \sqrt{\frac{2\times 5{,}000\times 49}{0.2\times 5.00}} = 700 \ \left(\textit{feasible}\right)$$

QD Example Solution (Cont.)

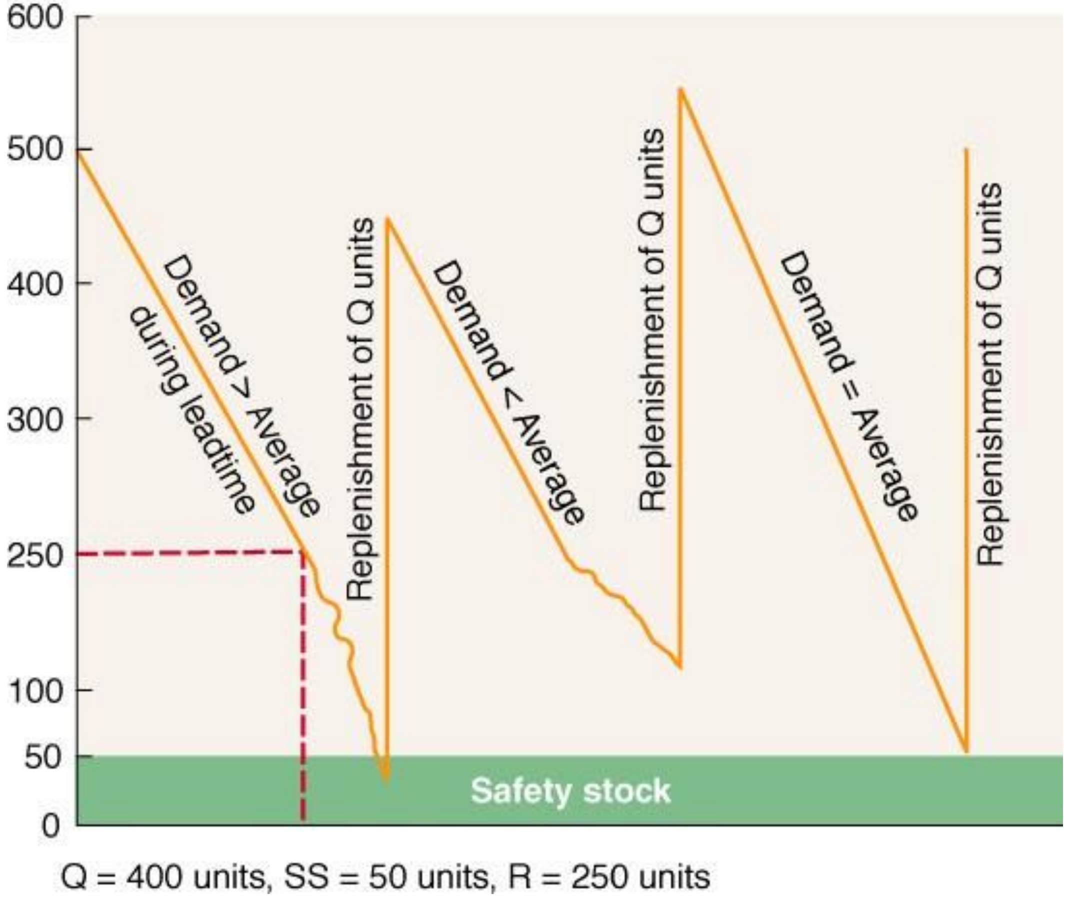
- Step 2

$$TC_{Q=700} = \frac{5,000}{700} \times 49 + \frac{700}{2} \times 0.2 \times 5.00 + 5.00 \times 5000 = \$25,700$$

$$TC_{Q=1000} = \frac{5,000}{1000} \times 49 + \frac{1000}{2} \times 0.2 \times 4.80 + 4.80 \times 5000 = \boxed{\$24,725}$$

$$TC_{Q=2000} = \frac{5,000}{2000} \times 49 + \frac{2000}{2} \times 0.2 \times 4.75 + 4.75 \times 5000 = \$24,822.50$$

What if Demand is Uncertain?



Safety Stock and Service Level

- Order-cycle service level is the probability that demand during lead time won’t exceed on-hand inventory.
- Risk of a stockout = 1 – (service level)
- More safety stock means greater service level and smaller risk of stockout

Safety Stock and Reorder Point

- Without safety stock:

$$R = dL$$

where R = reorder point in units
d = daily demand in units
L = lead time in days

- With safety stock:

$$R = dL + SS$$

where SS = safety stock in units

Reorder Point Determination

$$SS = z\sigma_{dL}$$

i.e.,

$$R = dL + z\sigma_{dL}$$

R = reorder point

d = average daily demand

L = lead time in days

z = number of standard deviations associated with desired service level

σ = standard deviation of demand during lead time

Safety Stock Example

- Daily demand = 20 units
- Lead time = 10 days
- S.D. of lead time demand = 50 units
- Service level = 90%

Determine:

1. Safety stock
2. Reorder point

Safety Stock Solution

Step 1 – determine z

$$\text{From Appendix B:} \quad z = 1.28$$

Step 2 – determine safety stock

$$SS = 1.28 \times 50 = 64 \text{ units}$$

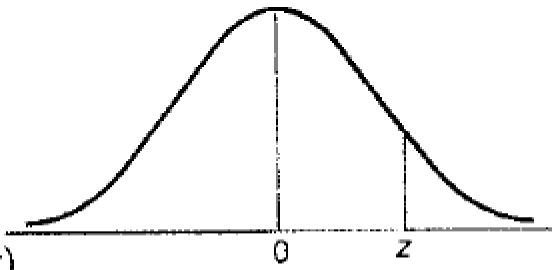
Step 3 – determine reorder point

$$R = dL + SS = 20 \times 10 + 64 = 264 \text{ units}$$

This table gives the area under the standardized normal curve from 0 to z, as shown by the shaded portion of the following figure.

Examples: If z is the standard normal random variable, then

$$\text{Prob}\ (0 \le z \le 1.32) = 0.4066$$
$$\text{Prob}\ (z \ge 1.32) = 0.5000 - 0.4066 = 0.0934$$
$$\text{Prob}\ (z \le 1.32) = \text{Prob}\ (z \le 0) + \text{Prob}\ (0 \le z \le 1.32)$$
$$= 0.5000 + 0.4066 = 0.9066$$
$$\text{Prob}\ (z \le -1.32) = \text{Prob}\ (z \ge 1.32) = 0.0934 \text{ (by symmetry)}$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767

ABC Inventory Classification

- **ABC classification** is a method for determining level of control and frequency of review of inventory items
- A Pareto analysis can be done to segment items into value categories depending on annual dollar volume
- **A Items** – typically 20% of the items accounting for 80% of the inventory value-use Q system
- **B Items** – typically an additional 30% of the items accounting for 15% of the inventory value-use Q or P
- **C Items** – Typically the remaining 50% of the items accounting for only 5% of the inventory value-use P

ABC Example: the table below shows a solution to an ABC analysis. The information that is required to do the analysis is: Item #, Unit $ Value, and Annual Unit Usage. The analysis requires a calculation of Annual Usage $ and sorting that column from highest to lowest $ value, calculating the cumulative annual $ volume, and grouping into typical ABC classifications.

Item	Annual Usage ($)	Percentage of Total $	Cumulative Percentage of Total $	Item Classification
106	16,500	34.4	34.4	A
110	12,500	26.1	60.5	A
115	4500	9.4	69.9	B
105	3200	6.7	76.6	B
111	2250	4.7	81.3	B
104	2000	4.2	85.5	B
114	1200	2.5	88	C
107	1000	2.1	90.1	C
101	960	2	92.1	C
113	875	1.8	93.9	C
103	750	1.6	95.5	C
108	600	1.3	96.8	C
112	600	1.3	98.1	C
102	500	1	99.1	C
109	500	1	100.1	C

Inventory Record Accuracy

- **Inaccurate inventory records can cause:**
  - Lost sales
  - Disrupted operations
  - Poor customer service
  - Lower productivity
  - Planning errors and expediting

- **Two methods are available for checking record accuracy**
  - Periodic counting-physical inventory
  - Cycle counting-daily counting of pre-specified items provides the following advantages:
    - Timely detection and correction of inaccurate records
    - Elimination of lost production time due to unexpected stock outs
    - Structured approach using employees trained in cycle counting